Simplified Sensorless Control for BLDC Motor, Using DSP Technology

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Abstract

This paper describes a simple way to control, in a sensorless way, a Brushless DC (BLDC) motor for electric vehicle applications. To control this machine it is generally required to count with a position sensor because the inverter phases, acting at any time, must be commuted depending on the rotor position. Encoders and resolvers have been used for sensing rotor position with respect to stator. These sensors, however, make the motor system more complicated and mechanically unreliable. In this paper, a simple solution is presented to determine the commutation sequence of a BLDC motor with a sinusoidal flux distribution. The method is based on a two phase current sensing and the determination of the back emf. For trapezoidal flux distributions the solution may be implemented with some minor changes. The main characteristic of this type of motor, fed with quasi-square-wave currents, is that it only needs a six position sensor, and only one current controller for its full torque control. In contrast, the sinusoidal current type, the angular position needs to be known at any moment in order to control each of the three phase currents. The solution proposed makes use of information contained in the back emf, calculating the six commutation points required. This method is only applicable while currents can be sensed, so it needs to be complemented with a starting method. The system was implemented using a fast digital signal processor (TMS320F241) which is programmed with a closed loop PI current control in order for the motor to produce a constant torque. Additionally, a fiber optic link is used between the controller and the inverter. This minimizes noise production and possibilities of error on commutations. As current transducers LEM sensors were used. The motor tested is a 12kW (16HP) Brushless Permanent magnet Motor, with an IGBT inverter with a commutation frequency of 15 kHz. Experimental results of the currents are shown. Copyright 2002 EVS19

Keywords: electric drive, brushless motor, control system, drive.

1. Introduction

In electric traction, like in other applications, a wide range in speed and torque control for the electric motor is desired. The DC machine fulfils these requirements, but this machine needs periodic maintenance. The AC machines, like induction motors, and brushless permanent magnet motors do not have brushes, and their rotors are robust because commutator and/or rings do not exist. That means very low maintenance. This also increases the power-to-weight ratio and the efficiency. For induction motors, flux control has been developed, which offers a high dynamic performance for electric traction applications [1,2]. However, this control type is complex and sophisticated. The development of brushless permanent magnet machines [3-5] has permitted an important simplification in the hardware for electric traction control. Today, two kinds of brushless permanent magnet machines for traction applications are the most popular: i) the Permanent Magnet Synchronous Motor (PMSM), which is fed with sinusoidal currents, and ii) the Brushless DC Motor (BDCM), which is fed with quasi-squarewave currents. These two designs eliminate the rotor copper losses, giving very high peak efficiency compared with a traditional induction motor (around 95 % and more in Nd-Fe-B machines in the 20 to 100 kW range). Besides, the power-to-weight ratio of PMSM and BLDC motor is higher than equivalent squirrel cage induction machines. The aforementioned characteristics and a high reliability control make this type of machine a powerful traction system for electric vehicle applications [6]. However, sensing the phase currents and the position of the rotor are two of the drawbacks that this type of machine have. In this paper, a control system for brushless DC motors, based on a DSP from Texas Instruments is proposed. The DSP used is the TMS320F241, which has been programmed to produce a simple way to control the machine currents, and to evaluate the instantaneous position of the rotor. The method is based on 1) the measurement of the currents, based on a common dc current I_{MAX} , which is obtained taking the absolute values of two of the three real phase currents, and 2) the calculation of the commutation instants based on the slope variations of this current I_{MAX} .

The most popular way to control BLDC motor for traction applications is through the use of voltagesource current-controlled inverters. The inverter must supply a quasi-square current waveform, whose magnitude, I_{MAX}, is proportional to the machine shaft torque [7]. Then, by controlling the phase currents, torque and speed can be adjusted. There are two ways to control the phase currents of a BLDC: 1) through the measurement of the phase currents, which are compared and forced to follow a quasi-square template, and 2) through the measurement of the dc link current, which is used to get the magnitude of the phase-currents, I_{MAX} . In the first case, the control is complicated, because it is required to generate three, quasi-square current templates, shifted 120° for the three phases. Besides, these current templates are not easy to follow for the machine currents, because of phase-shifts and delays introduced [8]. In the second case, it is difficult to measure the dc current, because the connection between transistors and the dc capacitors in power inverters are made with flat plates to reduce leakage inductance. Then, it becomes difficult to connect a dc current sensor. To avoid those drawbacks, the equivalent dc current is obtained through the sensing of two the three armature currents. From these currents, the absolute value is taken, and a dc component, which corresponds to the amplitude I_{MAX} of the original phase currents, is obtained. This dc component is then used to drive the BLDC motor. This solution, which was proposed in [9] using discrete (analog and digital) components, has been implemented now using DSP technology. Besides, all the PWM signals for the six transistors are transmitted from the control to the inverter through fiber optics.

The second issue to ve solved in this paper is the position estimator of the rotor. For PMSM motors many methods for obtaining rotor position and speed have been proposed in the literature. These methods are called observers and are based on mathematical models of the motor. They need an accurate knowledge of motor parameters, and some of them are: 1) The Luenberger Observer [10], which is one of the simplest, and has a linear feedback. It is of a deterministic type and does not take into account system and measurement noise. 2) The Sliding Mode Observer [11] that has a similar structure to the previous observer, but the error feedback signal is a non-linear switching function, and 3) The extended Kalman filter [12], which is a non-linear, recursive, stochastic filter, based on noise properties of systems. Its feedback gain is a function of the covariance matrices of the system and measurement noise. This last solution needs knowledge of noise properties and is computationally more intensive than the previous methods.

For BLDC motors, a precise determination of the rotor angle is not necessary. It is only required to know the position of commutation points, because the objective is to achieve quasi-square current waveforms, with dead time periods of 60°.

This paper has been divided into two main research areas; the first consists on the design and implementation of a current control system for the BLDC machine, and the second part describes the design of the logic, which calculates the instantaneous position of the rotor. All this calculations are realized with the DSP TMS320F241. However, the main contribution that this paper wants to show is the way the instantaneous position of the rotor is evaluated.

2. Current Control System

The circuit of fig.1 shows, in an equivalent hardware configuration, the basic topology of the current controller, which has been implemented for software inside the DSP. As was mentioned before, the DSP calculates the currents from two of the three phases, takes their absolute values and makes a process similar to a rectification inside the chip. Later on, this "rectified" current, called I_{MAX} , is compared with a reference coming from the accelerator or brake pedal, and the error signal is processed through a PI controller. The output of the PI controller is compared with a saw-teeth carrier signal, to generate the PWM for the power transistors. At the same time, the position estimator discriminates which couple of the six transistors of the inverter should receive this PWM signal. The information about direction (neutral, forward or reverse) is defined by the operator of the system.

The phase currents of figure 1 are sensed using LEMs (current sensors), whose output signals are in first place filtered by two independent analog, second order, lowpass filters: one for current control

and the other for position estimation. Then, this independent signals are passed through analog-todigital converters. The digital information of these two signals (current through I_{MAX} and position estimation through dI_{MAX}/dt) are then processed on the DSP. It is important to mention that all the operations displayed in figure 1, with exception of some conditioning hardware, are executed inside the DSP. Figure 2 shows the block diagram of the duties realized inside and outside the DSP. These characteristics give a great flexibility to the overall control system of the machine.



Figure 1: Hardware representation of the DSP controller, to manage the phase currents of the machine, and Digital Selector from position estimator

The I_{MAX} current, which carries information about the dI_{MAX}/dt in each commutation produced by the PWM, is used for estimation of the instantaneous position of the rotor. This procedure is explained in the next paragraph. However, it is very important to say that the position estimator system developed here becomes feasible because the PWM signal is the same for all the six transistors of the inverter. This fact will be clarified later on.



Figure 2: Control scheme based on DSP

3. Position Estimator System

The position estimator for a brushless dc machine needs to detect six positions, which determine the commutation points. The diagram of figure 3 shows an ideal commutation sequence for the trapezoidal emf of the machine, with their flat 120° maximums, centered with the phase-to-neutral currents for optimal commutation.



Figure 3: Ideal currents and induced voltages, and the commutation points for a BLDC motor.

- a) phase currents
- b) phase-to-neutral voltages
- c) phase-to-phase-voltages

As can be seen in figure 3, the slopes of both, phase-to-phase and phase-to neutral voltages change abruptly during commutation. Then, either of these voltages can be used for position detection. But, since the neutral point of the machine is floating and not accessible, the only way to use this information is through the phase-to-phase induced voltages. Looking at the phase-to-phase voltages of figure 3, the method proposed for commutation instances detection, is based on the fact that, for optimum operation, the commutation points of phase a (switching-on) are produced exactly at the point where the induced phase-to-phase voltage between phases b and c changes its slope from zero to a non-zero values very rapidly. The same happens for phases b and c respect to (*Egc-Ega*) and (*Ega-Egb*) respectively. But the problem is that the change in slope of these voltages and the voltages themselves are not measurable, and then a special strategy has to be used. In this kind of machine, with only two of the three phases conducting at any time, the circuit to be considered will include the two phases in conduction. The model for a BLDC motor being used is shown in the figure 4.



Figure 4: Motor Circuit Model

When a couple of phases of the motor are conducting, i.e. the current I_{MAX} flowing from phase *a* to phase *b*, the differential equation to be solved is:

$$V_{DC} = 2R \cdot I_{MAX} + 2L \cdot \frac{dI_{MAX}}{dt} + (Ega - Egb)$$
(1)

where V_{DC} is the dc supply voltage of the inverter, R and L are the stator winding resistance and inductance respectively, I_{MAX} the phase currents being controlled by the PWM, dI_{MAX}/dt the slope of I_{MAX} , and Ega-Egb, the phase-to-phase back emf. Similar equations can be written for the other two phase-to-phase induced voltages.

As the current is kept at the reference value I_{REF} by the controller, the term $2R \cdot I_{MAX}$ does not change significantly, and then:

$$2L \cdot \frac{dI_{MAX}}{dt} + (Ega - Egb) = V_{DC} - 2R \cdot I_{MAX} \approx K$$
⁽²⁾

where K=constant

$$\therefore \quad 2L \cdot \frac{d^2 I_{MAX}}{dt^2} \approx -\frac{d}{dt} (Ega - Egb)$$
(3)

Despite I_{MAX} is kept around a constant reference value, the differential terms of I_{MAX} does not disappear, because of the slopes. These slopes balance the instantaneous magnitude of the induced voltage (*Ega-Egb*). Then, the information about the commutation points for position estimation can be obtained from the slopes of I_{MAX} , avoiding the calculation of (*Ega-Egb*).

Now, because the current controller implemented here uses the same PWM for all the transistors, when two phases are in operation during their corresponding 60°, both the transistors commutate at the same time. Then, the positive slope of the resultant phase current always satisfies equations (2) and (3). For the negative slopes, there is only a change in the sign of these equations. Then, (2) and (3) allow determining the commutation points through the positive slopes of the current I_{MAX} . With the corresponding changes in the sign of these equations, the negative slopes can also be utilized for the calculations when the duty cycle of the PWM is smaller than 0,5. The figure 5 shows the only possible topologies during the conduction period of 60° mentioned before.



Figure 5: The two possible topologies during positive conduction of phase "a"

The commutation points are always on the opposite phase, that means, the sudden variation in slope of voltage (*Ega-Egb*) will give information to switch-on phase c, and so. However, by looking at equation (3) it is easy to realize that any change in (*Ega-Egb*) will produce an opposite change in the slope of dI_{MAX}/dt . Then it is not required to evaluate the *emf* voltages but only the changes in the slope of dI_{MAX}/dt . The sudden change in the slope of the voltage will also produce a sudden change in the slope of dI_{MAX}/dt , and this point will indicate the instant of commutation. In the case of sinusoidal *emf* (like the machine tested in our laboratory) the commutation points should be located 30° after the slope reaches the minimum value, or 30° after d^2I_{MAX}/dt^2 equals zero. After commutation is produced, a new couple of phases will be conducting, and they will give information for the next commutation.



Figure 6: Calculation of dI_{MAX}/dt, to get commutation instants in a BLDCM

The figure 6 shows how dI_{MAX}/dt variation is obtained from the positive slopes of the phase current, in agreement with eq. (2). The current controller of the machine maintains the phase-currents with a reference value I_{MAX} , and the slope of I_{MAX} is being computed (in this example) only when the PWM signal makes the current increase with a positive slope dI_{MAX}^+/dt . When a BLDC machine is being used, if a sudden change in the slope is produced, the corresponding phase is commutated as shown in figure 6 a). In the case of a PMSM (sinusoidal *emf*), when the minimum of dI_{MAX}/dt is reached, then a delay of 30° is applied to switch-on the current in the corresponding phase, as shown in figure 6 b). This also means 30° after $d^2I_{MAX}/dt^2=0$. The algorithm is applied sequentially to commutate the phases *a*, *b*, *c* and so on.

At this point, it is quite important to mention that the strategy proposed in this work, does not depend on the values of R and L, because is based on current slope variations (dI_{MAX}/dt) regardless of it's magnitude (I_{MAX}) , which is kept at the value of the reference. Even more, R and L can change in time because of many factors, but the slope variations used in this method will be produced always at the same places. The system neither uses the value of the slope. It uses the change of the slopes with time to detect the commutation points. However, if I_{MAX} does not remain around a flat reference, then the term $2R \cdot I_{MAX}$ has to be considered. In this case, eq (3) becomes:

$$\frac{d}{dt}\left(\frac{dI_{MAX}}{dt} + \frac{R}{L}I_{MAX}\right) = -\frac{d}{dt}(Ega - Egb)$$
(4)

which is equal to zero when (Ega-Egb) reaches its maximum value:

$$\frac{d}{dt}\left(\frac{dI_{MAX}}{dt} + \frac{R}{L}I_{MAX}\right) = 0$$
(5)

On its digital implementation an adequate time increment ΔT is considered (which depends on the linear model used to approximate dI_{MAX}/dt), then multiplying by this fix ΔT :

$$\frac{d}{dt} \left(\Delta I_{MAX} + \frac{R}{L} \Delta T \cdot I_{MAX} \right) = 0 \tag{6}$$

In this case, the term R/L must be known. However, a constant value for this parameter can be considered. In eq (6) ΔI_{MAX} is calculated from a set of samples taken form I_{MAX} on the time period ΔT . In order to find the desired minimum point of dI_{MAX}/dt , eq (6) is evaluated every time a PWM period finishes (with a frequency of 15 kHz).



Figure 7: Current sensing points to evaluate positive slopes.

Since the inverter tested was operated at a frequency of 15 kHz, in order to obtain phase current measurements that can give information about dI_{MAX}/dt , the sampling frequency must be faster than the 15 kHz. The DSP used has a minimum conversion time of 2µs which is equivalent to 500 kHz sampling rate. In order to allow time for calculations, samples are taken only for a period of 30 µs, either on the positive slope (conducting semi period of the PWM) or on the negative slope (nonconducting semiperiod). This situation will depend on the PWM duty cycle (bigger or smaller than 0.5 respectively). Special care has to be taken in order to avoid sampling currents on noise transient instants, like the ones produced just after a commutation has occurred. For this reason, a delay of 4 us is implemented before acquisitions are done. The figure 7 shows the instants where the samples are taken for the positive slope case. The current displayed here, is a real current from the machine.

One of the important features of this algorithm is that it needs the information of the corresponding phase current on the period of time before commutation is applied. This is not difficult to obtain because there are relatively clean signals, as was shown in figure 7.

To get a reliable information about the dI_{MAX}^+/dt , a linear statistical model was used to calculate the desired slope [13]. The general linear hypothetical model of full rank used is,

$$Y = X\beta + e \tag{7}$$

The form of the frequency function of the error (e) due to acquisition errors and noises unspecified, is assumed to have a zero mean,

$$E(e) = 0 \tag{8}$$

and a covariance matrix,

$$E(ee') = \sigma^2 I \tag{9}$$

Using the method of least squares, that is, finding the value of β where the sum of squares of *e* is minimum, the equation to minimize it is:

$$\sum_{i=1}^{n} e_{i}^{2} = e'e = (Y - X\beta)'(Y - X\beta)$$
(10)

The least-squares estimation of β is,

$$\hat{\beta} = S^{-1} X Y \tag{11}$$

where, *X* is the independent variable, *Y* the dependent and S = X'X.

According to the Gauss-Markoff Theorem, the best (minimum-variance) linear (linear functions of the y_i) unbiased estimation of β is given by the least squares. Solving for β as a function of the measured data,

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \frac{1}{n \sum (x_i - \overline{x})^2} \begin{pmatrix} \sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i \\ -\sum x_i \sum y_i + n \sum y_i x_i \end{pmatrix}$$
(12)

 β_2 is the slope of the line and β_1 the value of E(y) when x=0. An important fact of this result is that for fast calculations and a constant sampling rate (not necessarily uniform), β can be calculated as:

$$\beta = \left(S^{-1}X'\right)Y \tag{13}$$

Where the first term S⁻¹X' depends entirely on the independent values of x_i , which can be precalculated and be assumed as a constant vector. Then in the DSP, β is a simple vector product. This property makes real time calculations feasible.

As shown in figure 7, each sample is obtained averaging two points taken every two microseconds, and a total of six pairs of samples are taken to calculate the slope with the previous method. All the acquisition process takes just 28 μ s, which means, around 43% of the total period of the PWM signal. The problem is when the duty cycle of the inverter becomes smaller than 43%. In this case, the negative slope should be evaluated for position estimation. This possibility is not yet implemented, but there is no problem in doing it.

One of the drawbacks of the system is the impossibility to have information of rotor position when the rotor speed is too slow or the machine is not running. This problem is under study through the knowledge of inductance variation related with the position of the rotor.

4. Experimental Results

As our laboratory does not have a permanent magnet brushless machine with trapezoidal back *emf*, the experiments were done with a machine with sinusoidal *emf*. As seen previously, the proposed method can be implemented in this machine too, but with an increase in computation time and complexity. Characteristics of the machine are Inom= 140 A, Vdc= 120 Vdc, max speed=5000rpm, Magnes:Nd-Fe-B, weight=29 kg.

The oscillogram of figure 8 shows the phase currents of the machine, for a speed of 1000 rpm. The figure displays the currents in phases a and b, and the magnitude of I_{MAX} , which is 50 amps. The current variations during conduction are not very clear, but figure 7 gives some help in showing the cleanless of the slopes of the phase currents for position estimation purposes.





As the motor tested has sinusoidal *emf*, the second derivative of I_{MAX}^+ gives precise information of the switching instants by finding its zero crossing. The figure 9 a) shows the oscillogram of the $d/dt(I_{MAX}^+)$. It can be observed that the signal, obtained with the method explained in figure 6, is very noisy. However, with a digital filter and a logical saturator programmed inside the DSP, this signal can be cleaned to get a better waveform of $d/dt(I_{MAX}^+)$. The oscillogram of $d^2/dt^2(I_{MAX}^+)$ is shown in figure 9 b). It should be remembered that the into of commutation are delayed 30° with respect to the minimum (or 30° delayed when $d^2/dt^2(I_{MAX}^+)=0$). Then, the delay of 2 milliseconds in the digital filtering (see figure 9) means a maximum period of operation of the machine of 12 milliseconds. This

is traduced to a maximum frequency of 42 Hertz for the inverter. Higher frequencies will mean a delay in the switching instants. For this reason, is still under study the improvement in getting the position information to reduce the filter delay, to get position estimation for higher speeds.

The problem described above is for brushless motors with sinusoidal *emf*. However, for a real brushless dc motor this problem does not exist, because in this case the detection method is based on a hysteresis band and not on signal filtering. The figure 9 shows how the switching instant is obtained for a real brushless dc machine.



Figure 9. Oscillograms of:

a) dI_{MAX}/dt b) d^2I_{MAX}/dt^2 c) Ta^U , Tb^U and Tc^U

5. Conclusions

A different way to sense the phase currents and to control, in a sensorless way, a Brushless DC (BLDC) motor for electric vehicle applications, has been presented. The current is sensed taking the absolute value of two of the three phases, transforming this information in a dc current I_{MAX} , which is finally compared with a reference value from the accelerator pedal. The paper also presents a solution of the problem of the position estimation, which is based on the determination of the current slopes during the conduction periods. The solution proposed makes use of information contained in the slopes of the currents, calculating the six commutation points required for this kind of machine. The system was implemented using a fast digital signal processor (TMS320F241) which is programmed with a closed loop PI control for the phase currents. The processor also makes all the calculations required for position estimation. Additionally, the PWM signals are transmitted through a fiber optic link to minimize noise production and possibilities of error on commutations. Because of a lack of BLDC in laboratory, the motor tested was a 12kW (16HP) brushless permanent magnet motor, with sinusoidal back emf, which is fed with an IGBT inverter working at 15 kHz commutation frequency.

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7. Appendix

The specific linear model used for equations 7) to 12) was:

$$y_i = \beta_1 + \beta_2 x_i + e_i \tag{a}$$

Where β_i are unknown scalar constants and x_i are known. Referring to eq. (4) the matrices needed are:

$$X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \dots & \dots \\ 1 & x_n \end{pmatrix} \qquad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \qquad Y = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{pmatrix}$$
(b)

Then, calculating S,

$$S = X'X = \begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix}$$
(c)

and

$$S^{-1} = \frac{1}{n \sum (x_i - \bar{x})^2} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{pmatrix}$$
(d)

With the previous results, and replacing it on eq. (11), eq (12) is obtained.

8. References

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